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ANALYTICAL SOLUTION FOR FREQUENCY RESPONSE OF EQUIPMENT IN LATERALLY LOADED MULTISTOREY BUILDINGS

WEN-JENG HSUEH*,†
Department of Naval Architecture and Ocean Engineering, National Taiwan University, Taiwan, ROC

SUMMARY

An analytical and closed-form frequency response of equipment mounted on multistorey buildings subjected to horizontal ground motion is proposed. In this study, the dynamics of the equipment and the building is expressed as a state-flow graph model, in which the interaction effect between the equipment and the building is considered. Based on the graph model, the analytical results for the frequency response of the acceleration of the equipment and the internal force in the support are derived. One of the advantages of this method is that the closed-form solutions of the frequency response expressed by polynomial form will be easily examined by analytical and numerical computations without complex operation. Moreover, the dynamic of the primary and secondary systems and their dynamic interaction are expressed separately in the derived formula. Thus most of the items in the formula need not be computed repeatedly for different supports of the equipment in design. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: primary–secondary structure; equipment–structure system; dynamic interaction; frequency response; state-flow method

INTRODUCTION

Computation of the dynamic response of equipment attached to structures subjected to ground motion is vital for seismic design of the equipment and its support. Many pioneers in this research have studied the combined primary–secondary system for more than two decades. The decoupling method is the simplest way to analyse the dynamics of primary and secondary structure separately.[1,2] The influence of the dynamic interaction on the response of the secondary structure was first studied by Newmark.[3] A model approach based on the composite primary–secondary (P–S) system employing the notation of an effective mass ratio is investigated.[4] Later, the perturbation method was applied to calculate approximately the floor spectra or the response of the secondary system.[5,6] Another approximate scheme based on the modal synthesis and perturbation theory for the composite P–S system is also proposed.[7] The appropriate application of the perturbation theory requires a good understanding of relative orders of the mass ratio,

† Associate Professor
* Correspondence to: Wen-Jeng Hsueh, Department of Naval Architecture and Ocean Engineering, National Taiwan University, 73 Chow-Shan Road, Taipei, Taiwan, R.O.C.

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turning parameter and damping. An energy-based statistic method was utilized to study the coupling effect between the P and S structural system and to estimate the mean-square response of the secondary system.[8] Most of the previous studies are based on approximation technology. Recently, some exact solutions for various types of P–S systems have been proposed.[9,10] However, those methods are suitable for numerical computation since it is difficult to express the solution in analytical and closed form.

In this paper, the two-way state-flow graph method[11,12] is used for the dynamic analysis of this P–S system. The frequency response function of the equipment with respect to horizontal ground motion is directly formulated as an analytical and closed form according to the graph model. Finally, equipment attached to a seven-storey building is examined in the numerical example to demonstrate the advantage of this method.

GRAPH MODEL OF EQUIPMENT AND PRIMARY SYSTEMS

An equipment installed on a multistorey building subjected to horizontal ground motion is usually modeled as a single Degree-of-Freedom (DOF) secondary system attached to an N-DOF primary system as shown in Figure 1. Assume that the secondary system is fixed on the $\bar{i}$th unit of the primary system. The equations of motion of the secondary system subjected to periodical-base excitation can be expressed as

$$X_s = \frac{F_s}{m_s \omega^2} \tag{1}$$

$$F_s = (\mathrm{j} c_s \omega + k_s)(X_s - X_{\bar{i}}) \tag{2}$$

where m_s, k_s, and c_s are the mass, spring and damping constant of the secondary system. ω is the excitation frequency of the ground. j is equal to $\sqrt{-1}$. X_s, $X_{\bar{i}}$ and F_s are the Fourier transform of $x_s(t)$, $x_{\bar{i}}(t)$ and $f_s(t)$, which are the displacement of the mass m_s, the displacement at the attachment point, and the shear force in the interface between the secondary and the primary system. If F_s is assigned as the input and X_s is assigned as the output, the system dynamics expressed in equation (1) can be described by a state-flow graph as Figure 2(a). In a similar way, equation (2) can be represented as Figure 2(b). From Figures 2(a) and (b), we see that the input variable of Figure 2(a) is equal to the output variable of Figure 2(b) and the output variable of Figure 2(a) is also the same as one of the input variables in Figure 2(b). Figures 2(a) and (b) can be assembled to generate a new graph model for the secondary system as shown in Figure 3. Then, the input variable of the system is reduced to $X_{\bar{i}}$. The variables of the system X_s and F_s are functions of the input variable $X_{\bar{i}}$.

The primary system consists of N units of the subsystem, which includes a mass series connected to spring and damper. For each subsystem, unit i, except unit $\bar{i}$, the dynamic equations in the frequency domain are similar to equations (1) and (2). However, for the unit $\bar{i}$, since a reaction force generated by the secondary system also acts on the floor level of unit $\bar{i}$, the dynamic equation for mass $m_{\bar{i}}$ is replaced by

$$X_{\bar{i}} = -\frac{F_s + F_{\bar{i}+1} - F_{\bar{i}}}{m_{\bar{i}} \omega^2} \tag{3}$$

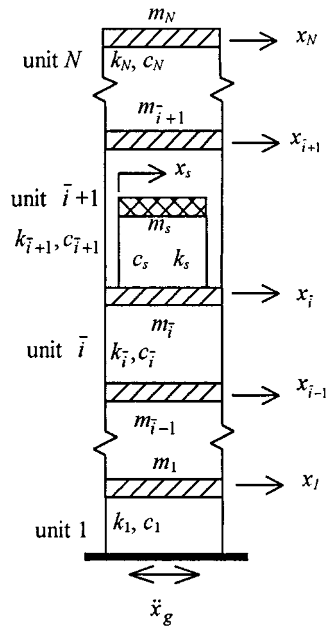

Figure 1. Schematic diagram of equipment attached to a multi-storey building

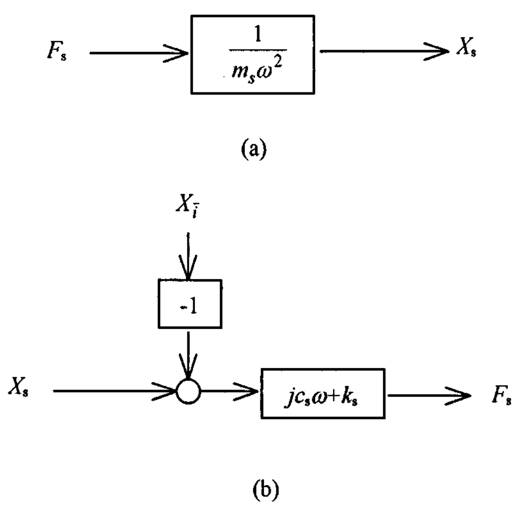

Figure 2. State-flow graph model for (a) equation (1), (b) equation (2)

Thus, the state-flow graph model for each unit of the building can be created. Then, the state-flow graph model of the total structure can be configured according to the arrangement of the structure as shown in Figure 4.

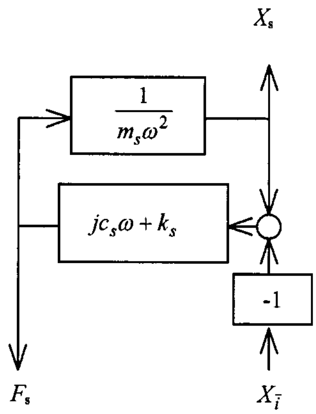

Figure 3. Two-way state-flow graph model of the secondary system

FREQUENCY RESPONSE FUNCTION

From the state-flow graph model as shown in Figure 4, we see that there is only one input from the ground displacement and only one output to the displacement of the secondary system. The complex frequency response function from input variable X_0 to output variable X_s, defined by $T_{0_{dis}, s_{dis}} = X_s/X_0$, can be calculated by the gain formula[13] given as

$$T_{0_{dis}, s_{dis}} = P_1 D_1 / D \tag{4}$$

where D is the determinant of the state-flow graph, P_1 is the path gain of the forward path from input to output, and D_1 is the cofactor of the forward path defined by the determinant of the state-flow graph formed by deleting all loops touching the forward path. There is only one forward path from the input, X_0, to the equipment displacement, X_s, for the response function computation. The forward path gain is

$$P_1 = \frac{jc_s\omega + k_s}{-m_s\omega^2} \prod_{i=1}^{\bar{i}} \frac{jc_i\omega + k_i}{-m_i\omega^2} \tag{5}$$

The number of all loops in the graph model is $2N + 1$. The determinant of the state-flow graph can be calculated individually for the loops related only to the primary system, D_p, and the loops related to the secondary and primary system, D_{s1} and D_{s2} expressed as

$$D = D_p + D_{s1} + D_{s2} \tag{6}$$

$$D_p = 1 + \sum_{i_1=1}^{2N-1} L_{i_1} + \sum_{i_2=3}^{2N-1} \sum_{i_1=1}^{i_2-2} L_{i_1} L_{i_2} + \cdots + \sum_{i_{N-1}=2N-3}^{2N-1} \cdots \sum_{i_2=3}^{i_3-2} \sum_{i_1=1}^{i_2-2} L_{i_1} L_{i_2} \cdots L_{i_{N-1}}$$
$$+ L_1 L_3 \cdots L_{2N-3} L_{2N-1} \tag{7}$$

$$D_{s1} = L_{s1} D_p \tag{8}$$

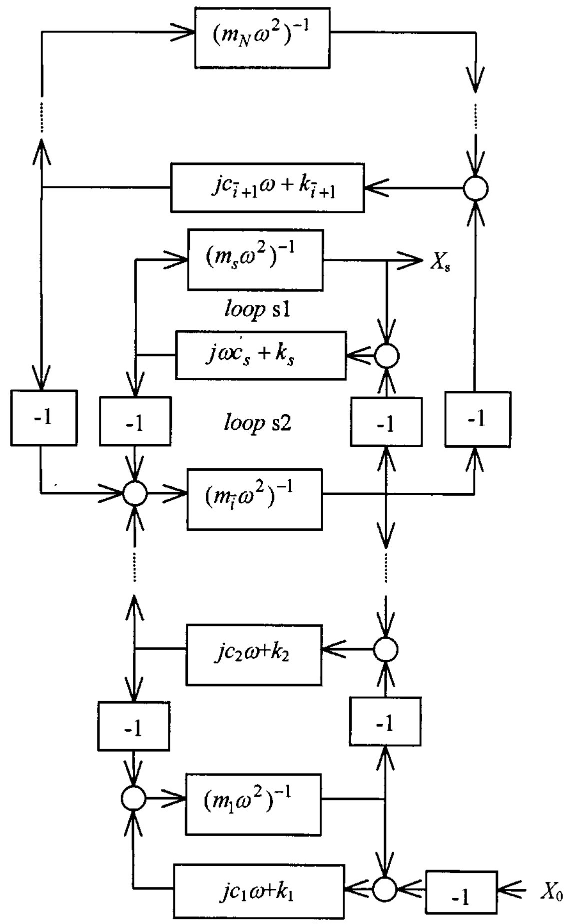

Figure 4. The graph mode of the equipment and primary structure

$$D_{s2} = L_{s2}\left(1 + \sum_{\substack{i_1=1\\ i_1\neq 2\bar{i},\, 2\bar{i}-1}}^{2N-1} L_{i_1} + \sum_{\substack{i_2=3\\ i_2\neq 2\bar{i},\, 2\bar{i}-1}}^{2N-1} \sum_{\substack{i_1=1\\ i_1\neq 2\bar{i},\, 2\bar{i}-1}}^{i_2-2} L_{i_1}L_{i_2} + \cdots + \sum_{\substack{i_{N-2}=2N-3\\ i_{N-2}\neq 2\bar{i},\, 2\bar{i}-1}}^{2N-1} \right.$$
$$\left. \times \sum_{\substack{i_2=3\\ i_2\neq 2\bar{i},\, 2\bar{i}-1}}^{i_3-2} \sum_{\substack{i_1=1\\ i_1\neq 2\bar{i},\, 2\bar{i}-1}}^{i_2-2} L_{i_1}L_{i_2}\cdots L_{i_{N-1}}\right) \tag{9}$$

where L_i is the negative of the ith loop gain for the primary structure from the bottom and L_{s1} and L_{s2} are the loop gain for loops s1 and s2 for the secondary, defined as

$$L_i = \begin{cases} \dfrac{\mathrm{j}\omega c_{(i+1)/2} + k_{(i+1)/2}}{-\, m_{(i+1)/2}\omega^2} & \text{for } i = 1,\ 3,\ 5,\ \ldots,2N-3,\ 2N-1 \\[2ex] \dfrac{\mathrm{j}\omega c_{i/2+1} + k_{i/2+1}}{-\, m_{i/2}\omega^2} & \text{for } i = 2,\ 4,\ 6,\ \ldots,2N-4,\ 2N-2 \end{cases} \tag{10}$$

$$L_{s1} = \frac{jc_s\omega + k_s}{-m_s\omega^2} \tag{11}$$

$$L_{s2} = \frac{jc_s\omega + k_s}{-m_{\bar{i}}\omega^2} \tag{12}$$

The cofactor of the forward path, D_1 is given by

$$D_1 = 1 + \sum_{i_1=2\bar{i}+1}^{2N-1} L_{i_1} + \sum_{i_2=2\bar{i}+3}^{2N-1} \sum_{i_1=2\bar{i}+1}^{i_2-2} L_{i_1}L_{i_2} + \cdots + \sum_{i_{N-1}=2N-3}^{2N-1} \cdots \sum_{i_2=2\bar{i}+3}^{i_3-2} \sum_{i_1=2\bar{i}+1}^{i_2-2} L_{i_1}L_{i_2} \cdots L_{i_{N-1}} + L_{2\bar{i}+1}L_{2\bar{i}+3} \cdots L_{2N-3}L_{2N-1} \tag{13}$$

The structure parameters of each unit of the primary structure in the function of L_i can be represented by the damping ratio, ξ_i and natural frequency, Ω_i, L_{s1} and L_{s2} are expressed by the natural frequency and damping ratio of the secondary structure, Ω_s and ξ_s. Thus, the frequency response of the displacement of the equipment to the displacement of the ground can be expressed by polynomial fraction:

$$T_{0_{dis}, s_{dis}} = \frac{\tilde{P}_1\tilde{D}_1}{-\tilde{\omega}^2\tilde{L}_s^{-1}\tilde{D}_p - \tilde{\omega}\rho_s\tilde{D}_{s2} + \tilde{D}_p} \tag{14}$$

where

$$\tilde{P}_1 = \tilde{L}_1 L_3 \tilde{L}_5 \cdots \tilde{L}_{2\bar{i}-3}\tilde{L}_{2\bar{i}-1} \tag{15}$$

$$\tilde{D}_1 = (-\omega^2)^{N-\bar{i}} + \left(\sum_{i_1=2\bar{i}+1}^{2N-1} \tilde{L}_{i_1}\right)(-\omega^2)^{N-\bar{i}-1} + \left(\sum_{i_2=2\bar{i}+3}^{2N-1} \sum_{i_1=2\bar{i}+1}^{i_2-2} \tilde{L}_{i_1}\tilde{L}_{i_2}\right)(-\omega^2)^{N-\bar{i}-2} + \cdots + \tilde{L}_{2\bar{i}+1}\tilde{L}_{2\bar{i}+3} \cdots \tilde{L}_{2N-3}\tilde{L}_{2N-1} \tag{16}$$

$$\tilde{D}_p = (-\omega^2)^N + \left(\sum_{i_1=1}^{2N-1} \tilde{L}_{i_1}\right)(-\omega^2)^{N-1} + \left(\sum_{i_2=3}^{2N-1} \sum_{i_1=1}^{i_2-2} \tilde{L}_{i_1}\tilde{L}_{i_2}\right)(-\omega^2)^{N-2} + \cdots + \tilde{L}_1\tilde{L}_3 \cdots \tilde{L}_{2N-3}\tilde{L}_{2N-1} \tag{17}$$

$$D_{s2} = (-\omega^2)^{N-1} + \sum_{\substack{i_1=1 \\ i_1 \neq 2\bar{i}, 2\bar{i}-1}}^{2N-1} \tilde{L}_{i_1}(-\omega^2)^{N-2} + \sum_{\substack{i_2=3 \\ i_2 \neq 2\bar{i}, 2\bar{i}-1}}^{2N-1} \sum_{\substack{i_1=1 \\ i_1 \neq 2\bar{i}, 2\bar{i}-1}}^{i_2-2} \tilde{L}_{i_1}\tilde{L}_{i_2}(-\omega^2)^{N-3} + \cdots + \sum_{\substack{i_{N-2}=2N-3 \\ i_{N-2} \neq 2\bar{i}, 2\bar{i}-1}}^{2N-1} \cdots \sum_{\substack{i_2=3 \\ i_2 \neq 2\bar{i}, 2\bar{i}-1}}^{i_3-2} \sum_{\substack{i_1=1 \\ i_1 \neq 2\bar{i}, 2\bar{i}-1}}^{i_2-2} \tilde{L}_{i_1}\tilde{L}_{i_2} \cdots \tilde{L}_{i_{N-1}}(-\omega^2) \tag{18}$$

$$\tilde{L}_i = \begin{cases} 2j\omega\xi_{(i+1)/2}\Omega_{(i+1)/2} + \Omega^2_{(i+1)/2} & \text{for } i = 1, 3, 5, \ldots, 2N-3, 2N-1 \\ \rho_{i/2+1}(2j\omega\xi_{i/2+1}\Omega_{i/2+1} + \Omega^2_{i/2+1}) & \text{for } i = 2, 4, 6, \ldots, 2N-4, 2N-2 \end{cases} \tag{19}$$

$$\tilde{L}_s = 2j\omega\xi_s\Omega_s + \Omega_s^2 \tag{20}$$

$$\rho_s = \frac{m_s}{m_{\bar{i}}} \tag{21}$$

$$\rho_i = \frac{m_i}{m_{i-1}} \tag{22}$$

In equation (14), $\tilde{P}_1$, $\tilde{D}_1$, $\tilde{D}_\mathrm{p}$ and $\tilde{D}_{\mathrm{s}2}$ are $\bar{i}$, $2(N-\bar{i})$, $2N$ and $(2N-2)$-order polynomial of ω. Moreover, we see that the term of $\tilde{D}_\mathrm{p}$ in equation (14) has no relationship to the equipment. $\tilde{P}_1$, $\tilde{D}_1$, and $\tilde{D}_{\mathrm{s}2}$ are dependent on the number of floor where the equipment is installed. Only both terms of ρ_s and $\tilde{L}_\mathrm{s}$ have a relationship with the weight of the equipment and supports. So, if the mass of the equipment and the floor of the building where the equipment to be installed have been determined, the dynamic response of the equipment will only be a function of $\tilde{L}_\mathrm{s}$.

In some applications, we might pay attention to the frequency response of the acceleration of the secondary system relative to the acceleration of the ground denoted by $T_{0_\mathrm{acc},\mathrm{s}_\mathrm{acc}}$. However, $T_{0_\mathrm{acc},\mathrm{s}_\mathrm{acc}}$ and $T_{0_\mathrm{dis},\mathrm{s}_\mathrm{dis}}$ are identical. The frequency of the shear force of the elasticity segment of the secondary system relative to the displacement of the ground denoted by $T_{0_\mathrm{acc},\mathrm{s}_\mathrm{force}}$ can be calculated based on equation (1) leading to

$$T_{0_\mathrm{acc},\mathrm{s}_\mathrm{force}} = -m_\mathrm{s} T_{0_\mathrm{acc},\mathrm{s}_\mathrm{acc}} \tag{23}$$

IDENTICAL PRIMARY SYSTEMS

If the structure properties of each unit of the primary structure are identical, the item $\tilde{L}_i$ for each i will be identical to $(2\mathrm{j}\omega\xi\Omega+\Omega^2)$. Then, the terms of $\tilde{P}_1$, $\tilde{D}_1$, $\tilde{D}_\mathrm{p}$, $\tilde{D}_{\mathrm{s}2}$ lead to

$$\tilde{P}_1 = (2\mathrm{j}\omega\xi\Omega+\Omega^2)^{\bar{i}} \tag{24}$$

$$\tilde{D}_1 = \sum_{k=0}^{N-\bar{i}} \frac{(2N-2\bar{i}-k)!}{(2N-2\bar{i}-2k)!\,k!}(2\mathrm{j}\omega\xi\Omega+\Omega^2)^k(-\omega^2)^{N-\bar{i}-k} \tag{25}$$

$$\tilde{D}_\mathrm{p} = \sum_{k=0}^{N} \frac{(2N-k)!}{(2N-2k)!\,k!}(2\mathrm{j}\omega\xi\Omega+\Omega^2)^k(-\omega^2)^{N-k} \tag{26}$$

$$\begin{aligned}\tilde{D}_{\mathrm{s}2} = &\sum_{k=0}^{N-1} \frac{(2N-2-k)!}{(2N-2-2k)!\,k!}(2\mathrm{j}\omega\xi\Omega+\Omega^2)^k(-\omega^2)^{N-1-k}\\ &+\sum_{k=0}^{\bar{i}-2} \frac{(2\bar{i}-3-k)!}{(2\bar{i}-3-2k)!\,k!}(2\mathrm{j}\omega\xi\Omega+\Omega^2)^{k+2}(-\omega^2)^{N-3-k}\end{aligned} \tag{27}$$

NUMERICAL EXAMPLES

Equipment installed on a seven-storey building is considered in the numerical calculation to illustrate the application of the present method. The mass, damping and stiffness properties of the first floor of the primary structure are given as: $m_1 = 2{\cdot}4\times10^5$ kg, $c_1 = 2{\cdot}2\times10^5$ N-m/sec, $k_1 = 4{\cdot}55\times10^8$ N/m. The structural properties of the other floors are the same: mass $= 2{\cdot}0\times10^5$ kg, damping $= 2{\cdot}0\times10^5$ N-sec/m, stiffness $= 3{\cdot}5\times10^8$ N/m. The natural frequency and damping ratio of unit 1 and other units are 43·5 rad/sec and 0·010 and 41·8 rad/sec and 0·012, respectively.

In the first example, equipment weighing 20 kg supported by three different supports with various natural frequencies, $\Omega_\mathrm{s} = 20{\cdot}9$, $41{\cdot}8$, $83{\cdot}6$ rad/sec ($\Omega_\mathrm{s}/\Omega_4 = 0{\cdot}5$, 1, 2), and the same damping ratio, 0·012, on the fourth floor is considered. The acceleration response ratio of the equipment, defined as the amplitude ratio of the acceleration response of the equipment to the

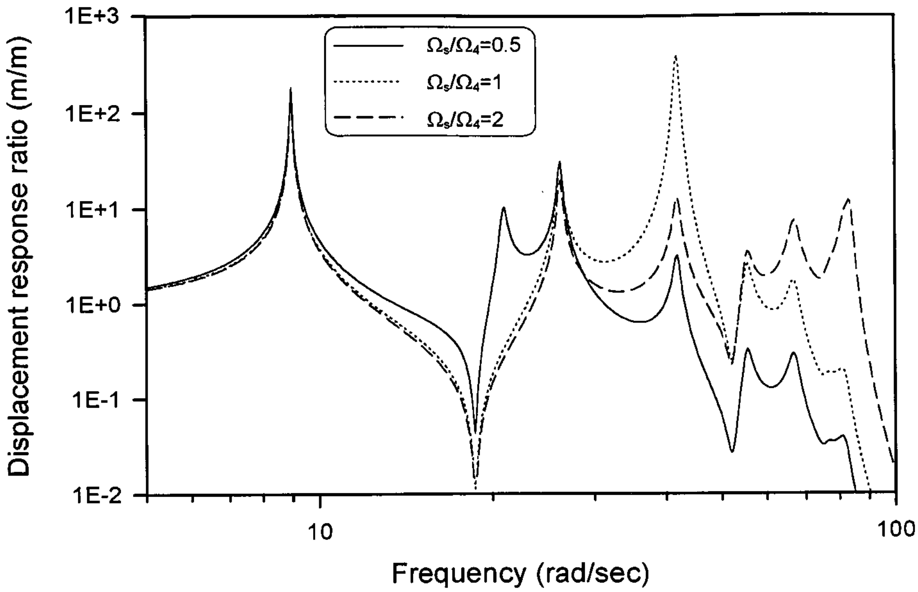

Figure 5. Response ratio of the equipment at floor 4, $m_s/m_4 = 0{\cdot}01$, $\xi_s = 0{\cdot}012$

acceleration of ground motion, is the absolute function of the frequency response, which can be calculated directly from the derived formula of equation (14). Since only the term of $\tilde{L}_s$ is different for various supports of the equipment, one calculation of the other terms $\tilde{P}_1$, $\tilde{D}_1$, $\tilde{D}_p$, $\tilde{D}_{s2}$ and ρ_s is sufficient. In the equation, the terms of $\tilde{P}_1$, $\tilde{D}_1$, $\tilde{D}_p$ and $\tilde{D}_{s2}$ are 4, 6, 14 and 12 order polynomial. Those terms can be expressed by non-dimensional form, defined $\tilde{\omega} = \omega/41{\cdot}8$ rad/sec, as

$$\tilde{P}_1 = 3{\cdot}27\times10^7\tilde{\omega}^4 - 5{\cdot}46\times10^{-5}\mathrm{j}\tilde{\omega}^3 - 3{\cdot}43\times10^{-3}\tilde{\omega}^2 + 9{\cdot}56\times10^{-2}\mathrm{j}\tilde{\omega} + 1 \quad (28)$$

$$\begin{aligned}\tilde{D}_1 = & -\tilde{\omega}^6 + 1{\cdot}20\times10^{-1}\mathrm{j}\tilde{\omega}^5 + 5{\cdot}00\tilde{\omega}^4 - 2{\cdot}87\times10^{-1}\mathrm{j}\tilde{\omega}^3 - 6{\cdot}00\tilde{\omega}^2 \\ & + 7{\cdot}17\times10^{-2}\mathrm{j}\tilde{\omega} + 1\end{aligned} \quad (29)$$

$$\begin{aligned}\tilde{D}_{s2} = & -\tilde{\omega}^{12} - 2{\cdot}59\times10^{-1}\mathrm{j}\tilde{\omega}^{11} - 1{\cdot}09\times10^1\tilde{\omega}^{10} + 2{\cdot}14\mathrm{j}\tilde{\omega}^9 + 4{\cdot}48\times10^1\tilde{\omega}^8 \\ & - 6{\cdot}27\mathrm{j}\tilde{\omega}^7 - 8{\cdot}76\times10^1\tilde{\omega}^6 + 7{\cdot}84\mathrm{j}\tilde{\omega}^5 + 8{\cdot}22\times10^1\tilde{\omega}^4 - 3{\cdot}89\mathrm{j}\tilde{\omega}^3 \\ & - 3{\cdot}25\times10^1\tilde{\omega}^2 + 5{\cdot}50\times10^{-1}\mathrm{j}\tilde{\omega} + 1\end{aligned} \quad (30)$$

$$\begin{aligned}\tilde{D}_p = & \tilde{\omega}^{14} + 3{\cdot}07\times10^{-1}\mathrm{j}\tilde{\omega}^{13} + 1{\cdot}29\times10^1\tilde{\omega}^{12} - 3{\cdot}08\mathrm{j}\tilde{\omega}^{11} - 6{\cdot}46\times10^1\tilde{\omega}^{10} \\ & + 1{\cdot}14\times10^1\mathrm{j}\tilde{\omega}^9 + 1{\cdot}60\times10^2\tilde{\omega}^8 - 1{\cdot}92\times10^1\mathrm{j}\tilde{\omega}^7 - 2{\cdot}01\times10^2\tilde{\omega}^6 \\ & + 1{\cdot}44\times10^1\mathrm{j}\tilde{\omega}^5 + 1{\cdot}20\times10^2\tilde{\omega}^4 - 3{\cdot}87\mathrm{j}\tilde{\omega}^3 - 2{\cdot}70\times10^1\tilde{\omega}^2 + 1{\cdot}67\times10^{-1}\mathrm{j}\tilde{\omega} + 1\end{aligned} \quad (31)$$

It is very easy to compute the value of the terms $\tilde{P}_1$, $\tilde{D}_1$, $\tilde{D}_p$, $\tilde{D}_{s2}$ and ρ_s for each frequency. For $\Omega_s = 20{\cdot}9$, $41{\cdot}8$, $83{\cdot}6$ rad/sec, $\tilde{L}_s$ is $1{\cdot}20\times10^{-2}\,\mathrm{j}\omega + 0{\cdot}25$, $2{\cdot}39\times10^{-2}\mathrm{j}\omega + 1$, and $4.78\times10^{-2}\mathrm{j}\omega + 4$, but $\tilde{P}_1$, $\tilde{D}_1$, $\tilde{D}_p$, $\tilde{D}_{s2}$ and ρ_s do not change. Then, the acceleration response ratio can be obtained from the absolute value of equation (14) as shown in Figure 5.

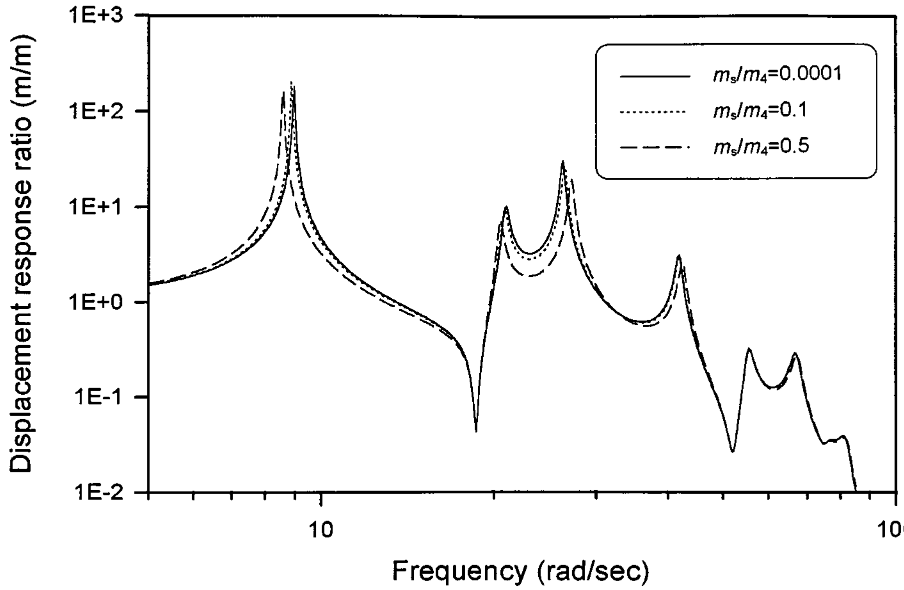

Figure 6. Response ratio of the equipment at floor 4, $\Omega_s/\Omega_4 = 0{\cdot}5$, $\xi_s = 0{\cdot}012$

In the second case, equipment with three varying weights, 20, 2000 and 10 000 kg, with the same natural frequency and damping ratio of the equipment 20·9 rad/sec and 0·012, are examined. In the formula equation (14), only the item ρ_s is altered (10^{-4}, 10^{-2}, 0·5) for different weights of equipment. The acceleration response ratio can be obtained from equation (14) and the previous results of $\tilde{P}_1$, $\tilde{D}_1$, $\tilde{D}_p$, $\tilde{L}_s$ and $\tilde{D}_{s2}$, are shown in Figure 6.

CONCLUSIONS

A state-flow graphic model has been proposed for the dynamic analysis of the equipment mounted on multistorey buildings, in which the dynamic interaction is considered. Based on the graph model, the complex frequency responses of the equipment subjected to horizontal ground motion are derived. Since the results are closed form and expressed as polynomial with a summation type coefficient, it is very easy to examine by analytical and numerical procedures. Moreover, the truncation error in computation can be reduced because the derived results are exact. For the uniform primary structure, the response is expressed as a simple and perspicuous form. From the numerical examples, we see that only one term in the formula is changed if the weight or the support of the equipment is altered. Most of the terms of the formula do not need to be computed repeatedly for different conditions, indicating that this method is suitable for the design of attached equipment.

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REFERENCES

1. J. M. Biggs and J. M. Roesset, 'Seismic analysis of equipment mounted on a massive structure', in R. J. Hansen (ed.) *Seismic Design of Nuclear Power Plants*, MIT Press Cambridge, MA, 1970.
2. M. P. Singh, 'Generation of seismic floor response spectra', *J. Engng. Mech. Div. ASCE* **101**, 593–607 (1975).
3. N. M. Newmark, 'Earthquake response analysis of reactor structures', *Nuclear Engng. Des.* **20**, 303–322 (1972).
4. J. L. Sackman and J. M. Kelly, 'Seismic analysis of internal equipment and components in structures', *Engng. Struct.* **1**, 179–190 (1979).
5. A. Der Kiureghian, J. L. Sackman and B. Nour-Omid, 'Dynamic analysis of light equipment in structures: response to stochastic input', *J. Engng. Mech. Div. ASCE* **109**, 90–110 (1983).
6. T. Igusa and A. Der Kiureghian, 'Generation of floor response spectra including oscillator-structure interaction', *Earthquake Engng. Struct. Dyn.* **13**, 661–676 (1985).
7. L. E. Suarez and M. P. Singh, 'Floor response spectra with structure-equipment interaction effects by a mode synthesis approach', *Earthquake Engng. Struct. Dyn.* **15**, 141–158 (1987).
8. G. Chen and T. T. Soong, 'Energy-based dynamic analysis of secondary systems', *J. Engng. Mech. ASCE* **120**, 514–534 (1994).
9. G. Chen and T. T. Soong, 'Exact solutions to a class of structure-equipment systems', *J. Engng Mech. ASCE* **122**, 1093–1100 (1996).
10. L. E. Suarez and M. P. Singh, 'An exact component mode synthesis approach', *Earthquake Engng. Struct. Dyn.* **16**, 293–310 (1988).
11. W. J. Hsueh, 'Analysis of vibration isolation systems using a graph model', *J. Sound Vib.* **216**, 399–412 (1998).
12. W. J. Hsueh, 'On the vibration analysis of multi-branch torsional systems', *J. Sound Vib.* (in press).
13. S. J. Mason, 'Feedback theory — further properties of signal flow graphs', *Proc. IRE* **44**, 920–926 (1956).

